THE INTERNATIONAL JOURNAL OF CREATIVITY & PROBLEM SOLVING
2014, 24(2), 101-122

Using a Creativity Model to Solve The Place-value Problem in Kindergarten

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A creativity model based on paired constraints was used to solve a core problem in early American math education, place-value. To create the solution, one set of constraints precluded specific elements in existing math curricula. The other promoted substitutes, including an explicit base-10 count and a single manipulative, a count-and-combine chart that visually represented the base-10 decimal system and promoted highly-focused practice in combining tens and ones in single- and double-digit addition and subtraction. The new curriculum was piloted for the entire school year. At pre-test, there were no differences between children in the pilot and control (regular curriculum) groups. At post-test, the pilot group outperformed the control not only on place-value, but also on single- and double-digit addition and subtraction, and number line estimation. The evidence suggests that creativity models can make significant contributions to solving problems in early education.

INTRODUCTION

The term place-value is self explanatory. The placement of each digit in a multi-digit number determines the number's place value. In a two-digit number, the values of the two places are tens and ones, with the tens represented by the digit on the left and the ones represented by the digit on the right. The place-value problem is this. American children mistake, for example, the two ones digits in the number they call "eleven" (11) as being of equal value. Chinese, Japanese, and Korean children, who call the same number "ten-one," do not make that mistake (Miura & Okamoto, 2003). In consequence, they outperform American children not only on place-value, but also on multi-digit addition and subtraction (Fuson & Kwon, 1992; Song & Ginsberg, 1987; Stigler, Lee, & Stevenson, 1990).

The proposed solution (to the place-value problem, and by extension, to the multi-digit addition and subtraction problems) was developed using a problem-based creativity model. The solution was structured in what is called a problem space (Newell & Simon, 1972). A problem space has three parts: an initial state, a goal state, and between the two, a search space for creatively constructing a solution path. The construction is creative because the path is new. In the model, constraint pairs are used to structure

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Thanks to my enthusiastic, energetic, extremely effective lab assistants Aviva Hamavid, Alex Hinton, and Elena Mayer. Special thanks to Emil Carafa, the principal who permitted, indeed encouraged, this program to be implemented in his school, and to Catherine Tronza, the kindergarten teacher without whose help it would not have been implemented so successfully.

the path (Reitman, 1965; Simon, 1973; Stokes, 2006, 2010, 2013a). One of each pair precludes something specific in the initial state; the other directs or promotes search for a substitute. The process has been called solution by substitution (Stokes, 2012, 2013b, 2014).

Creating the Solution

The goal was not simply to introduce an explicit base-10 counting system (count for short), but to embed it in a curriculum that taught children to think mathematically: in patterns and structures, using numbers and symbols to represent the patterns and structures. The name of the new curriculum – *Only the NUMBERS count©* -reflects its goal. Table 1 shows the problem space.

Table 1
Problem Space for New Math Curriculum

Problem Parts	Description		
Initial State	Current curricula		
Search Space	Constraint pairs		
	Preclude		*Promote*
	English language count	→	Explicit base-10 count
	Multiple manipulatives	→	Single manipulative
	Non-numeric	→	Numbers, symbols, patterns
	Split practice	→	Deliberate practice
Goal State	New curriculum		
Criterion	Thinking in numbers, symbols, and patterns		

The initial state was current curricula, characterized by components in the preclude column of the search space. The goal state was a new curriculum, characterized by substitutions in the promote column of the search space. I consider each preclude-promote pair in turn.

Solution by Substitution 1: The Explicit Base-10 Count

The primary constraint pair precluded the English count, and promoted in its place an English language version of the Asian (i.e., Chinese, Japanese, and Korean) counts. The differences between the two – one concealing, the other clarifying the recursive base-10 patterning of the number system - account in large part for the American child's problem with place-value (Fuson, 1990; Miura & Okamoto, 2003). The differences are readily apparent in Table 2, which compares the English and the Asian counts from 10 through 39.

There are several important things to notice. First, in the Asian count, there are only ten number names (1 to 10) that combine to form the higher, two-digit numbers. The combination algorithm is itself apparent in the count. Second, *ten* appears in every number name for the Asian count above ten in the table, but only once for the English. Just look at 11. The English number name *eleven* does not suggest place-value, whereas the Asian number names, which translates to *ten-one,* specifies it. Third, the order of the Asian number names parallels that of the marks. *Ten-six* has the same place-value ordering as the written marks *16*. The English name *sixteen* reverses the order, putting the single digit 6 in the 10's place. This occurs for the numbers 14 through 19, with the names for the numbers 11 through 13 further diverge from the place-value.

Table 2
English and Asian (Chinese, Japanese, Korean) Number Names

Numbers	Names		Numbers	Names		Numbers	Names	
	English	Asian		English	Asian		English	Asian
10	*ten*	*ten*	20	twenty	two-*ten*	30	thirty	three-*ten*
11	eleven	*ten*-one	21	twenty one	two-*ten*-one	31	thirty one	three-*ten*-one
12	twelve	*ten*-two	22	twenty two	two-*ten*-two	32	thirty two	three-*ten*-two
13	thirteen	*ten*-three	23	twenty three	two-*ten*-three	33	thirty three	three-*ten*-three
14	fourteen	*ten*-four	24	twenty four	two-*ten*-four	34	thirty four	three-*ten*-four
15	fifteen	*ten*-five	25	twenty five	two-*ten*-five	35	thirty five	three-*ten*-five
16	sixteen	*ten*-six	26	twenty six	two-*ten*-six	36	thirty six	three-*ten*-six
17	seventeen	*ten*-seven	27	twenty seven	two-*ten*-seven	37	thirty seven	three-*ten*-seven
18	eighteen	*ten*-eight	28	twenty eight	two-*ten*-eight	38	thirty eight	three-*ten*-eight
19	nineteen	*ten*-nine	29	twenty nine	two-*ten*-nine	39	thirty nine	three-*ten*-nine

As a consequence of their counts, American children think of numbers as collections of 1s; Asian children think of them as multi-unit structures of 10s and 1s (Fuson, 1990). If you call a number (21) "twenty-one," you will think of it as 21 ones. If however, you learn to call it "two-ten-one," you will think of it as 2 tens and a one. Thinking the second way, place-value will not be a problem.[1]

Solution by Substitution 2: The Single Manipulative
Manipulatives are organizational tools that physically represent problem structures (Zydney, 2008). The most effective ones "capture key structural features of desired internal representations and map onto them in transparent ways" (Siegler & Ramani, 2009, pa. 547). Important too are the consistent, long-term use of *specific* manipulatives (Sowell, 1989) with "*salient* features that suggest the correct meanings, do not possess misleading features and are *linked* over a *sustained* period to the target mathematical words and written marks" (Fuson and Burghardt, 2003, p. 299).

The italics are mine. To focus attention over a *sustained* period on the *salient* features of our number system and their *linkages* to place-value and single- and multi-digit addition and subtraction, a *specific*, single manipulative (the count-and-combine chart) was used throughout the school year. The hope was that the chart with its moveable parts, would, like the abacus, make numbers tangible, concrete things with visible patterned relationships among themselves. The relationships, along with the chart, were expanded as the school year progressed.

The first count-and-combine chart (shown in Figure 1) externalized numeric-symbolic relationships for numbers 1 through 10.

[1] This core difference becomes evident when children are asked to represent numbers using base-10 blocks [for a review, see Miura & Okamoto, 2003]. Most first graders in China, Japan and Korea represent multi-digit numbers with tens and ones blocks; most American children only use ones blocks (Miura, Kim, Chang, & Okamoto, 1988; Miura, Okamoto, Kim, Steere, & Fayol, 1993).

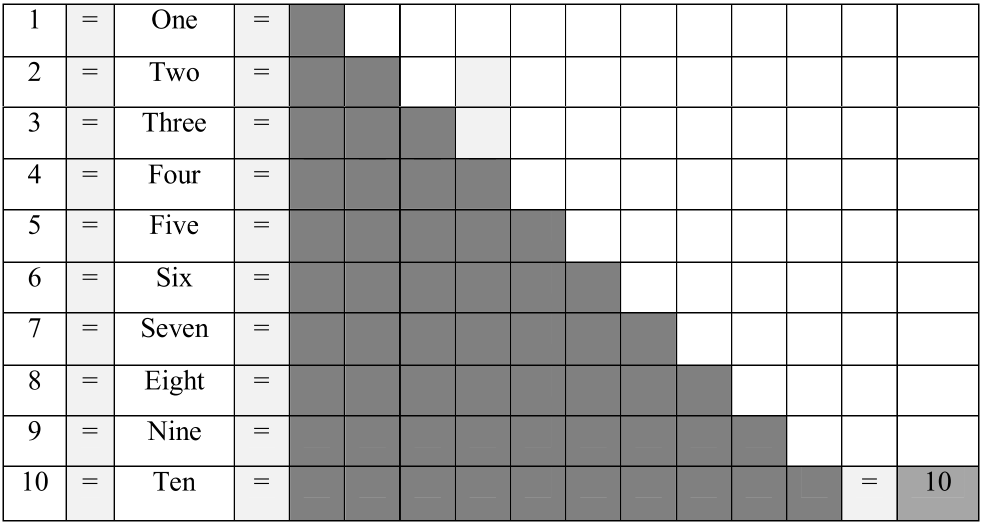


Figure 1. First Count-and-Combine Chart: One to Ten.

Notice the number of times the equals sign is shown. This was to ensure that children learned that it means "is the same as" or "equals" or "is equal to" rather than the common misinterpretations among American school children, who "read" the equals sign as meaning "adds to," "makes," "the answer," or "the end is coming" (Knuth, Stevens, McNeil, & Alibali, 2006; McNeil & Alibali, 2005; Rittle-Johnson & Alibali, 1999; Seo & Ginsburg, 2003).

Numbers, symbols, and "blocks" were made of velcro-backed laminated poster board. Children interacted visually (seeing the patterns), verbally (reciting each row out loud), and tactilely (moving the parts to produce "combinations") with the large class-size chart. The word *combination* is used purposively: numbers are combinations of other numbers.

Recitation took the following form:

Number one same as word one equals one block.
Number two same as word two equals two blocks.
Number three same as word three equals three blocks…

Figure 2 shows how children manipulated the "blocks" (on the chart or from baskets of blocks on their tables) to make combinations for three.

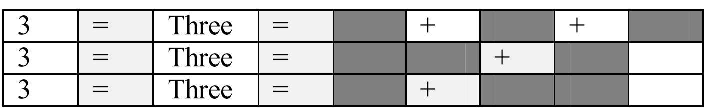


Figure 2. Addition Combinations for 3.

On the first line, three "equals" or is "the same as" one block plus one block plus one block. On the second, it is "equal to" two blocks plus one block. The third line reverses the second, one block plus two blocks. Children called this a “flip.”[2]

Figure 3 shows the second chart for numbers 10 through 20 (two-ten).

10	=	Ten	=	10										
11	=	Ten-one	=	10										
12	=	Ten-two	=	10										
13	=	Ten-three	=	10										
14	=	Ten-four	=	10										
15	=	Ten-five	=	10										
16	=	Ten-six	=	10										
17	=	Ten-seven	=	10										
18	=	Ten-eight	=	10										
19	=	Ten-nine	=	10										
20	=	Two-ten	=	10	10									

Figure 3. Second Count-and-Combine Chart: Ten to Two-ten.

Notice that ten is represented by a single block marked with the number 10. This is to emphasize that 10 is a *unit in itself,* not just a grouping of 10 ones. Notice too that 20, which is called two-ten, is represented by two 10 blocks. The next number 21, which is called two-ten-one, would be represented by two 10 blocks and one 1 block. The children did not use the 10 to 20 chart until they knew all the combinations (with one plus sign) for numbers up to, and including, 10. In reciting the chart, children always used the *Only the NUMBERS count©* base-10 names. For example, the second line was recited,

> *Number ten-one same as word ten-one equals one 10 block and one 1 block.*

Solution by Substitution 3: The Strictly Mathematical

Experts differ from novices in their ability to perceive and solve problems using large meaningful patterns and structures in their areas of expertise (Ericsson, 2006; Newell & Simon, 1972). For example, the patterns for writers are represented by words; for musicians, pitches. For mathematicians, the patterns are represented by numbers and symbols. With practice, and practice primarily with numeric-symbolic patterns. I hypothesized that young children could learn to think and problem solve like mathematicians. There was another reason for this focus on the strictly mathematical. The literature on analogy shows that transfer depends on recognizing similarities in elements or structures (Fuchs, Fuchs, Prentice, Burch, Hamlett, Owen, Hosp, & Jancek,

[2] There are actually four combinations for 3. The fourth is shown on the original count chart – 3 blocks together. More generally, for any given natural number, there are 2^{n-1} natural number sum combinations. Thus, with n = 3, we have $2^{3-1} = 2^2 = 4$ combinations.

2003; Gick & Holyoak, 1980, 1983; Holland, Holyoak, Nisbett, & Thagard,1986; Holyoak & Thagard, 1999). In this view, without an existing mathematical structure, there is nothing onto which a child can transfer a word problem. Word problems were therefore postponed to late in the school year.

Solution by Substitution 4: Deliberate Practice

By split practice, I meant switching between kinds of problems and/or materials. For example, the control class (which used *New Jersey Mathematics: Scott Foresman-Addison Wesley)* "worked on" numbers through 5 early in the school year. Working included: reading a math story and counting the objects in the story, playing a number game with a spinner and markers, making groups with tiles, and matching written numbers with groups of objects. "Working on" did not include addition until May, nor subtraction until June. In between, students worked on miscellaneous topics, including measurement, time, money, and shapes.

In contrast, lesson plans in the pilot were based on what is called "deliberate practice" (Ericsson, 2006; Ericsson, Krampe, & Tesch-Romer, 1993). Deliberate means focused on specific aspects of a skill to be developed in highly varied ways. Variation here means *switching between solutions* rather than between kinds of problems or materials. Practice with the chart and the moveable blocks was iterative and elaborative. Children practiced the pattern of the base-10 count by reciting and reconstructing each chart. They practiced the structures of base-10 solutions for addition and subtraction. As the numbers increased, so did the number of possible combinations. For example, there are 4 combinations for three (3, 2 + 1, 1 + 2, 1 + 1 + 1), 8 for four (4, 3 + 1, 1 + 3, 2 + 2, 2 + 1 + 1, 1 + 1 + 2, 1 + 2 + 1, 1 + 1 + 1 +1), 16 for five, and so on.

In principle, this sort of focused, incremental, deliberate practice is remarkably similar to a Japanese first grade curriculum described by Murate and Fuson (2006) as "coherent, with fewer topics that build over the year," and contrasted with the "mile wide inch deep US pattern" (p. 454).

Research Questions

Research questions focused on the overall effectiveness of the pilot curriculum as well as the contributions of each core component: the explicit base-10 count, the single manipulative, and deliberate practice. To evaluate on-going effectiveness, both the pilot class and a control class using the district curriculum were observed once a week. To evaluate cumulative effectiveness, mathematical performance at the start (pre-testing) and the end (post-testing) of the school year between the two classes was assessed. I predicted that, on the post-test, the pilot class would out-score the control on place-value, single- and double-digit arithmetic and subtraction, and perhaps on number line estimation.

METHOD

Participants

Forty-five kindergarteners in two classes at a suburban public school served as participants. The children were not placed in homogeneous groups by ability. Rather, sorting was done by gender to equalize the number of boys and girls in each class. One teacher and class were randomly chosen to be the pilot group; the other, to be the control group. Both groups adhered to the New Jersey State Standards in math. The pilot group, like the control, used materials from Scott Foresman-Addison Wesley

(Math Series Copyright 2008) to cover the following topics: Time, Money, Patterns, Directionality, Graphs/Charts, Fractions, Geometry, and Measurement. The intervention replaced materials covering numbers and numeric relations. The time for math was allotted equally across groups. Descriptions of each group its respective teacher follow.

Pilot Group. At the start of the school year there were 23 students (11 female, 12 male) in the pilot group. Of these, 13 students were Hispanic, 5 were White, and 5 were Asian; 7 were classified as ESL, 7 as economically disadvantaged. By the end of the year, there were 20 students: three males (1 White, 2 Hispanic) had relocated. Mean age at the start of the school year was 6 years, 5 months; range was 6 years to 7 years, 8 months.

The teacher for the pilot group participated in the development of the new curriculum. Teacher and experimenter met once a week. To ensure fidelity of treatment, the experimenter and three research assistants also observed the math lesson on the meeting day. While the core elements were pre-planned, the timing and fine tuning of their implementation depended on the teacher's judgment of the children's comprehension and thus their readiness to move on to more advanced material. The teacher had previously used both *Everyday Math©* (for two years at another school) and the district's current curricula. She had four years total experience, all of them teaching kindergarten.

Control Group. At the start of the school year, there were 20 students (10 female, 10 male) in the control group. Of these, 9 were Hispanic, 8 were White, 2 were Black, and 1 was Asian; 3 were classified as economically disadvantaged, 5 as ESL. By the end of the year, there were 2 additional students: 1 female (Black) and 1 male (Asian). Mean age at the start of the school year was 6 years, 5 months; range was 5 years to 7 years, 7 months

The teacher for the control group followed the district's curriculum. She had eleven years experience teaching, all at Washington School. She taught kindergarten for ten years, and first grade for one.

Procedure And Materials

The study was conducted in three phases. Phase 1 involved pre-testing to assess children's prior knowledge. Phase 2 involved piloting the base-10 intervention, as well as observing on a weekly basis both the pilot and the control classes. Phase 3 involved post-testing to assess what had been learned. Testing was conducted by the primary experimenter and three undergraduate research assistants (2 female, 1 male).

Phase 1: Assessing Prior Knowledge. Initial evaluations took place at the start of the school year. There were two reasons for the pre-test. One was to assure that the two classes were homogeneous in mathematical ability and/or preparation before the pilot intervention. The second was to compare specific changes in numeric understanding after the intervention.

Table 3 summarizes tasks on the pre-test. Children were tested on their ability to count and to recognize numbers and symbols. Both groups were expected to do fairly well on these tasks. They were also tested on their understanding of place-value, and their ability to locate numbers on a number line. The place-value task was a variant of the Choose the Larger Number Test, which requires choosing the larger of a pair of numbers (Fuson & Briars, 1990). The child was first asked to name the number (e.g., 16). Repeating the name given by the child, the experimenter then asked "If this is sixteen, which number is bigger, the first or the second?"[3] Children pointed to, rather than circled, the larger digit. An answer was scored "correct" only if a child identified the larger digit as a ten. With 16, for example, the child might say "the first number is really a 10." For 25, the correct answer would identify the first digit (the 2) as two-tens or twenty.[4]

Table 3
Pre-Test Tasks.

Category	Content
Counting	Children were asked to count as high as they could. Counting was coded as correct up to the first error (If a child counted 11, 12, 15, her score was 12, the highest number correctly counted).
Number and symbol identification	
	Children were asked to read aloud 10 written numbers (1, 2, 4, 5, 7, 8, 12, 15, 20, 32) and three symbols (plus, minus, equals) presented in problem format (e.g., 2 + 2 = 4). Correct responses for the + sign were: plus, and *N* more, add. Correct responses for the = sign were equals or same as. Correct responses for the – sign were: minus, less, and take away.
Place-value	Children were asked (a) to read aloud the written numbers 16, 25, and 31; (b) tell the experimenter which of each pair was bigger and (c) explain their answers.
Number line.	Children were asked to locate the numbers 2, 3, 25, 6, 80 and 67 (in that order) on six separate number lines. Each line was marked with a 0 at one end and a 100 at the other. There were no other marks on the line. Before asking the child to locate each number, the experimenter said, "Remember, this is a number line, if 0, a little number, is here (pointing) and 100, a big number, is here (pointing), where does *N* go?"

Phase 2: Piloting the program. The core materials – the explicit base-10 count, the count-and-combine chart, and deliberate practice focused on strictly mathematical skills – have already been described. Figure 2 showed how children manipulated the blocks to create addition combinations for the number three. This section shows how they made addition combinations above 10. Appendix A demonstrates using the blocks to do subtraction.

Breaking Apart the Ones. Figure 4 shows how the children initially used the blocks to create addition combinations for the number *ten-three*. To make these combinations,

[3] Since 16 is a two-digit number, the correct terminology is actually "which digit is bigger?" However, since the children were not familiar with the term digit, the question was asked using "number."

[4] An alternative place-value test using tens and one blocks (Miura, Kim, Steele, & Fayol, 1993) could not be used because it replicated what the pilot group would be doing all year.

they broke apart the three 1 blocks. This was not difficult, since they already knew the combinations for 3.

13	=	Ten-three	=	10										
				10	+									
				10	+		+		+					
				10	+		+							
				10	+			+						

Figure 4. Combinations for Ten-three, Breaking Apart Only the Ones.

Breaking apart the tens. Next, the teacher had them break apart the tens. To show this efficiently, I use written marks instead of blocks. The children did both, first breaking apart the 10 and 1 blocks, then writing out their combinations with marks. Recall, as stated above, they did not use the 10 to 20 chart until they knew all the addition combinations (with 1 plus sign) for numbers up to and including 10.

Starting with 13 = 10 + 2 + 1 (the last line of Figure 4), the teacher said, "Can we make this longer?" which here meant breaking apart the tens and the ones.

Breaking the 10 into 7 + 3, the problem became 13 = 7 + 3 + 2 + 1.

Breaking the 7 into 6 + 1, the problem became 13 = 6 + 1 + 3 + 2 + 1.

Breaking the 6 into 4 + 2, the problem became 13 = 4 + 2 + 3 + 1 + 2 + 1.

Continued on, the problem eventually broke down into

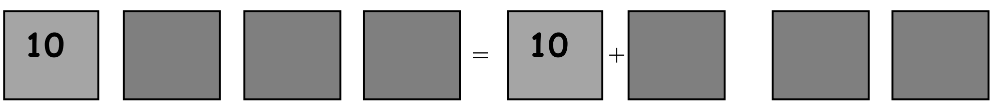
13 = 1+ 1 +1 +1 + 1 +1 +1 +1 +1 +1 +1 +1 +1.

Notice that the exercise allowed children at all levels to participate successfully: the faster ones broke apart the higher numbers (like the 10); the slower ones broke apart the lower numbers (like the 3s and 2s). The faster children also demonstrated their mastery of combinations by breaking up the numbers without manipulating the blocks.

Recombining the 10s and 1s. The children now recombined numbers they had broken apart. For example, starting with 13 = 7 + 3 + 2 + 1,

If 7 and 2 were combined, the problem became 13 = 9 + 2 + 1.

If 9 and 2 were combined, the problem became 13 = 11 + 2 which, arranged with loose blocks on their tables, would look like this



… and which would be said *ten-three equals ten-one plus two.*

Notice how each of the above examples reinforces the idea that *numbers are combinations of other numbers* .Notice too how the use of the 10 blocks and the explicit base-10 names reinforces the idea of *place-value*.

Phase 3: Assessing New Knowledge. The post-test (see Table 4) administered at the end of the school year included old, expanded and new sections.

RESULTS

Since this study involved a small number of students, I present descriptive statistics for pilot and control groups at pre- and post-testing. In the discussion section, I include data from statewide computerized tests given three years after children participated in the study.

Table 4
Post-Test Tasks

Category	Content
Counting	Identical to the pre-test.
Number and symbol identification	Identical to the pre-test.
Place-value.	Two problems (56, 11) were added to the original 3.
Number line.	One number (50) was added to the original 6.
Addition	Children solved two single digit (3 + 5, 6 + 2) and two double digit (12 + 4, 21 +11) problems. They were asked (a) to read the problem, (b) solve it, and (c) tell the experimenter how they did it.
Subtraction	Children solved two single digit (5-3, 7-5), and three double digit (10 – 6, 10 – 10, 22 - 12) problems. They were asked (a) to read, (b) solve, and (c) explain their method to the experimenter.
Combinations	Children read three numbers (8, 12, 23) aloud and were asked to make up an arithmetic problem (e.g., __ + ___ = 8) for each. Children who made up combinations were asked how they figured each one out.

Pre-test Post-test Comparisons

In this section, I compare performance in the pilot and control classes on items appearing on both pre- and post-tests. For one of these items (number line estimation), I include results reported by Siegler and Mu (2008) for Chinese students of comparable ages. Table 5 presents average accuracies on three of these: counting, identifying numbers and symbols, and place-value.

Both groups improved on the first two items. At pre-testing, the control group was more accurate on both count and number/symbol identification. At post-testing, the pilot group was more accurate on number-symbol identification; the groups were comparable on counting. However, counting correctly only indicates knowing order, not place-value.

Poor performance on place-value by children in the control group demonstrates this. None correctly identified the number with the greater value. For all numbers except 11, most (63%) picked the "bigger number" that "you count up to" regardless of its place. For the number 11, all said that the two "ones" were the same or equal. In contrast, all children (100%) in the pilot group correctly identified the first of each double-digit numeral as larger *because* it was a ten. Two examples illustrate their understanding.

Table 5

Pre- And Post-test Scores: Average Accuracies

	Measure											
	Count to 100				Number-Symbol Identification				**Place-value**			
	Pre		Post		Pre		Post		Pre		Post	
Group	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
Pilot	12.39	22.32	98.35	22.31	59.43	27.31	98.45	5.67	.0026	.01	**100.00**	.00
Control	22.85	28.82	93.45	20.91	67.55	19.01	88.95	9.10	.0000	.00	**.00**	.00

Note: Accuracy is expressed as proportion correct

For the number 31, one child explained: “The first is always bigger, it’s 3 tens.” For the number 11, another said: “The first 1 is a ten, the other is only a one.”

Figure 5 presents pre-test and post-test log and linear median magnitude estimates for both groups on the number line estimation task.

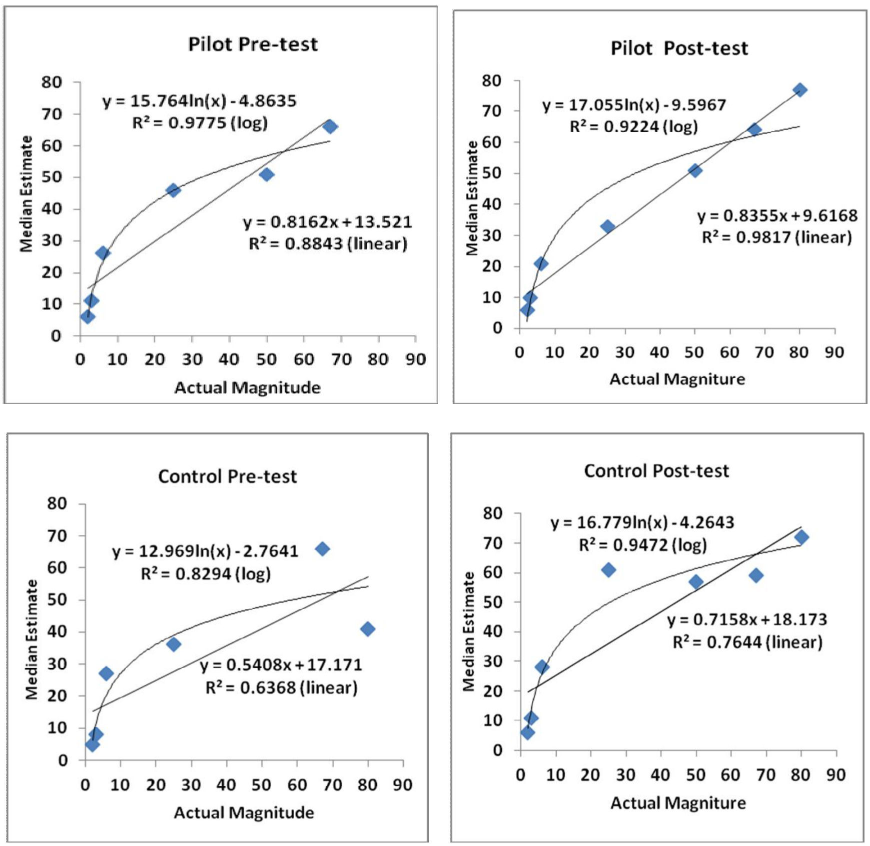


Figure 5. Pre- and Post-tests for Number Line Estimation.

At pre-test, variance in the group estimates for each number was better accounted for by logarithmic rather than linear functions. By post-testing, variance in the group estimates that was accounted for by the best fitting linear equation (R^2) increased substantially in the pilot group: from .8843 to .9817. This result is comparable to that reported for Chinese children (.95) of the same age group (mean ages at time of testing: Chinese, 5 years, 8 months; American, 5 years, 6 months) who generally out-perform American children on number line estimation (Siegler & Mu, 2008). The control group improved only slightly in their linear estimates (R^{2s} from .6368 to .7644); the variance in their post-test results was better accounted for by a log (R^2 = .9472) rather than a linear (R^2 = .7644) equation. This result is similar to that reported for American kindergarteners (log R^2 = .90) by Siegler & Mu (2008).

Post-test Comparisons. Five different problem types only appeared on the post-test: single and double digit addition, single and double digit subtraction, and addition combinations. Table 6 presents average accuracies on these items. In both groups, accuracies were higher on single vs. double digit addition and subtraction problems. However, on all items, the pilot group performed far more accurately than the control.

Table 6

Post-test Scores: Average Accuracies

	Measure									
	Addition				Subtraction				Addition Combos	
	Single Digit		Double Digit		Single Digit		Double Digit			
Group	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Pilot	97.50	11.18	36.65	26.42	95.00	15.38	66.80	21.73	86.70	27.35
Control	54.54	48.57	7.54	17.59	18.18	36.33	7.59	20.46	24.22	34.44

Note: Accuracy is expressed as proportion correct

The pilot group outscored the control on all measures, and more so when the problems involved double digits. Dramatic differences appeared in the group's understanding of subtraction. For single digit subtraction problems, 45% of the pilot group said they took the second number away from both sides of the subtraction sign. For example, with the problem 5 – 3 a typical explanation was: "I took 3 away from the 5 and the 3." As shown in Appendix A, this is how subtraction was taught. For double digit problems, 46% said they "knew" the answer. In contrast, 37% of children in the control group used addition in the single digit problems, 59% added in the double digit problems.

Differences in creating addition combinations were not surprising. Only the pilot group had experience with this task. For single digit combos, 65% of these children reported using "doubles" (e.g. 4 + 4 = 8); for the double digit combos, 60% used "the ten blocks" (e.g., 10 + 2 = 12, or 10 + 10 + 3 = 23).

DISCUSSION

The aim of this study was to evaluate the effectiveness of an early math curriculum designed to teach children to think and problem solve in base-10 and, by extension, the creativity model used to design the curriculum. Overall effectiveness was confirmed in the correspondence of results and predictions: kindergarteners in the pilot group outperformed those in the control group on place-value, single- and double-digit addition and subtraction, and number line estimation.

Why did the new curriculum work so well? To answer, I review the contributions of the core components and the paired constraint model that created them. I then consider two possible confounds, and answer three critical questions before offering conclusions.

Contributions of the Core Components

Contribution of the Count: Thinking in base-10

The explicit base-10 count facilitated mastery of place-value and by extension, multi-digit calculation. This is because the explicit count named the place of each number. For example, 26 is called "two-ten-six." Children using this kind of count *think* of 26 as two 10s and a 6. Thinking this way makes place-value obvious.

Thinking this way is also how Wegerif, in a recent interview, described the result of "speaking the language of mathematics" as becoming, in part, "a mathematician and thinking the world as a mathematician would think it" (Shaughnessy, 2014a, p. 41).

Contribution of the Count-and-Combine Chart: Externalizing base-10

Like the abacus, but simpler, the count-and-combine charts were designed to make *abstract* base-10 relations/patterns *visual* and *concrete.* Like the abacus with its moveable beads, the charts with their moveable "blocks" made numbers and symbols tangible, things in themselves, with clearly patterned relationships among themselves. All charts embodied the base-10 patterning of the number system. All were structurally similar, the numbers combining regularly in step-wise fashion, visualizing the repetitive patterns in the count and the combinations. Importantly, the 10 block – which represented *10 as a unit*, rather than as a group of 10 ones - externalized place-value per se.

Contribution of Deliberate Practice: Expertise in base-10

Deliberate practice is highly focused and highly variable. The focus was base-10 patterns and relations. The practice was iterative and elaborative. Children practiced the *pattern of the base-10 count* by reciting and recreating each chart. They practiced the *structure of base-10 solutions* for addition and subtraction problems. As a result of such practice, they became *experts,* pattern-seekers and problem-solvers in base-10.

Contribution of the Creativity Model

That American children have problems with place-value is not surprising. The problem has long been attributed to irregularities in the English language count that conceal rather than clarify the recursive base-10 patterning of the number system (Fuson, 1990; Fuson & Kwon, 1992; Geary, Bow-Thomas, Liu, & Siegler, 1996; Miura, Okamoto, Kim, Steere, & Fayol, 1993). What is surprising is that, while successful short-term interventions using, for example, base-10 blocks (Fuson & Briars, 1990; Miura & Okamoto, 2003) or "Egyptian" hieroglyphs (Baroody, 1987) have been reported, this is the *first study* involving early immersion in an English language version of the Asian (Chinese, Korean, Japanese) counts that make the base-10 patterning explicit. Why?

My suggestion is that the experts - math educators, in particular the makers and marketers of math programs and packages - are stuck in “successful” solutions (Stokes, 2008, 2014). Success here is, in large part, commercial. A representative from a major publisher told me that *Only the NUMBERS count©* could not be profitable without adding a lot of “bells and whistles.” Indeed, all the most recent curricula I examined, or observed in classrooms, retained *all* the hallmarks of the preclude column in Table 1, including the English language count, multiple manipulatives, split practice, videos with cartoon characters and work sheets with word problems related to the stories, etc. As a result, they are, to large degrees, revisions or elaborations of earlier programs. Their substitutions are not significant.[5]

The repetition mechanism, I submit, is simple: operant conditioning. It works this way. Experts are rewarded for reliably solving particular kinds of problems. They then (like the rest of us) reproduce, rather automatically, reliable responses that have “worked” in similar situations. These responses include activation of relevant knowledge structures or schema (Finke et al, 1992). As a result the variations they produce follow what Ward (1994; Ward, Patterson, & Sifonis, 2004) has called the “path of least resistance:” new ideas are structured in conventional ways. The same process occurs, perhaps even to a greater degree, with groups of experts who tend to rely on knowledge structures that are shared (Larson, Foster-Fishman, & Keys, 1994) and that confirm the group’s shared beliefs (Schultz-Hardt, Jochims, & Frey, 2002). In short, experts get stuck.

The paired constraint model is a tool for getting un-stuck. It counteracts the repetition/reliability mechanism by precluding search for substitutions in routinely visited parts of a problem space (the usual suspects), and promoting search in atypical, unfamiliar parts. The solution process is incremental. The customary elements of an existing “solution” (the initial state) are listed in the preclude column. One specific element is precluded, and its substitute identified. For the solution path (the promote column) to be unconventional and new, the substitution must be uncommon (at least in its context) and thus, unexpected. The unfamiliar will, in turn, direct search for subsequent substitutions.

Possible Confounds

Two things must be considered here: practice effects and teacher effects.

Practice Effects

There are two kinds of practice effects: one has to do with *amount* of practice, the other with *kind* of practice. Appendix B, which presents a month-to-month comparison of when numeric concepts were introduced in the pilot and the comparison classrooms, allows us to compare amount of practice. Children in the pilot group were adding numbers as early as September and subtracting by February; those in the control group did not do addition until May or subtraction until June. Kind of practice, specifically, the core elements of the pilot intervention, made early introduction of single- and multi-digit addition and subtraction possible. Children cannot practice what they cannot do. Thus, kind of practice, although different, is not separable from amount of practice.

[5] Another result is that, as related to me by different teachers using different programs, math packages come with more materials than can possibly be used in the time allotted to teach math.

Teacher Effects
There are actually two confounds here. First, the pilot teacher helped develop as well as implement the new curriculum. Second, while both teachers used the district's curriculum during the previous school year, children were not tested at the end of that year. Thus, we could not compare their relative effectiveness when using the same curriculum. However, the pilot teacher repeatedly said that the new curriculum was easier than the old, both for her to teach and for her students to learn.

To more accurately assess teacher effects, the curriculum was recently introduced in three kindergarten classes in the same public school. None of the teachers helped develop the curriculum. All used the same materials (count, charts, lesson plans). Here, I report preliminary data from this study. District-wide computerized math testing (*Renaissance STAR Math*) took place at the sixth month of the school year. In two classes, 93% of the children scored above grade level: in both classes, scores ranged from 0.5 (fifth month of kindergarten) to 2.3 or 2.4 (third or fourth month of second grade). In the third class, 87% of children scored at or above grade level: the range was 0.4 (fourth month kindergarten) to 1.8 (eighth month of first grade). The similarities here indicate that the pilot results were due primarily to the curriculum rather than to the individual teaching it.

Critical Questions

Three questions must be answered before conclusions can be made. The first concerns the standard American count. The second inquires "what comes next?" The third raises a challenge: how new, it asks, is the new curriculum.

What about the Standard "American" Count?
The expanded version of this question is: did the children in the pilot group know and use the standard count? The answer is yes. Kindergarteners in the pilot group were fluent in both the explicit base-10 count and the standard count. For example, they could interchangeably refer to the number 20 as "two-ten" or "twenty." This should not be surprising: fluency in a second language is directly related to the age at which an individual is immersed in the language (Johnson & Swain, 1997; Oyama, 1976, 1978). Being fluent in two languages includes recognizing the context in which to use one or the other. The children used the base-10 count when doing math, and the standard count in other contexts (e.g., dates, ages). Kindergarten, it appears, is an ideal time for children to become fluent in more than one count.

What Comes Next?
This is actually two questions. One concerns curriculum development. The other concerns performance when the pilot group children were exposed to the district curriculum. Expansion of the curriculum. During the 2014-2015 school year, the curriculum will be expanded into pre-k and second grade. Pre-K teachers will use the very early lessons plans developed for first grade. The second grade curriculum introduces a quite modified multiplication table that the children will use for both multiplication and division. Blocks will continue to be used, in novel configurations that show the patterns involved in multiplication and division. A different nomenclature will be used for fractions. For example, instead of calling 1/3 "one third," children will learn to call it "one of three equal parts." The multiplication materials have already been piloted with an "honors" math group made up of the 5 fastest math students in each of three first grade classes. The group met once a week in the spring.

Post-pilot Performance. Children in the pilot group were not transitioned to the district curriculum until 2nd grade. The class remained intact; first grade lesson plans, meant to follow introduction of the program in kindergarten, were developed in their classroom.

One answer to "what came next" came from their 2nd grade teacher, who reported that (1) children from the pilot class were (her word) amazingly fluent with numbers and patterns, and (2) that they had already mastered the district's 2nd grade materials. She moved them on to more difficult material.

The other answer came from district-wide testing during third grade. To see if the pilot curriculum had lasting effects on mathematical performance, district-wide computerized test scores (third grade, fifth month) for students from the original test and control groups were examined. The groups did not remain intact. By third grade, only 60% of the pilot group, and 72% of the control group were still in the school, and were distributed in three different classes.

Mean and median scores for the pilot group were both 4.3 (third month, fourth grade); scores ranged from 3.4 (fourth month, third grade) to 5.3 (third month, fifth grade). Mean and median scores for the control group were both 3.9 (ninth month, third grade); scores ranged from 3.2 (second month, third grade) to 4.5 (fifth month, fourth grade).

Overall, the performance gap was four months. The more dramatic difference appeared at the top: 44% of the pilot group scored at or above 4.5 (fourth grade, fifth month – a year ahead); only 8% of the comparison group did as well.

What came next, I would like to think, was the product of learning, and learning very early in skill acquisition, to think in numbers, symbols, and patterns.

How "New" Is the New Curriculum?

The questioner pointed out that the curriculum is based on existing (e.g, not new) East Asian approaches to number names and numeracy. This is true. Nonetheless, a curriculum designed to teach math must use the elements (including a count) that define the domain. Such defining elements I refer to as "source constraints." When developing something new to the domain, they are the things that one works against/ precludes or works with/promotes (Stokes, 2006; 2012). In fact, the effectiveness of the curriculum is based, in large part, on the count, which was the first and most significant substitution (for the American count) in the constraint model that produced the curriculum.[6]

The count is not new. The substitutions are.

The problem solving model used to construct the curriculum came from outside the domain; so too the problem solver, me. I had no stake in current theory or practice. This made it easy for me to preclude major elements in American curricula, and substitute in their places elements (the count, the single manipulative, deliberate practice) that, combined, created a *new* solution path to a *new* goal – having children think mathematically, in

[6] I have a confession to make regarding the count. I worked in Tokyo for several years and became fluent in the Japanese count. It was a necessity. I needed it on a daily basis to solve the problems of shopping, discovering when a train left Tokyo Eke, etc. It was, and is, efficient, and more important, easy. It was the first thing I thought of when I was asked to think about a new way to teach math. The papers I quote regarding the count were found after I already knew about the count.

numbers and symbols and the patterned relationships between them.[7]

The truly surprising thing to me and, I suspect, to the reviewer who raised the novelty question, is that it all worked so well.

CONCLUSION: IMPLICATIONS AND APPLICATIONS

The Pilot Program: Implications

The pilot demonstrates that early exposure to a curriculum using an explicit base-10 count, a single manipulative, and a pedagogy focused on deliberate, strictly mathematical practice, can help very young children master – and enjoy mastering – mathematical material (place-value, linear estimation, double-digit addition and subtraction) not required by the new Common Core Standards at their grade level. Early is important because later achievement in mathematics is strongly related to earlier achievement (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Bodovski & Farkas, 2007; Classens, Duncan, & Engel, 2008; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Williamson, Appelbaum, & Epanchin, 1991). In the 2014-2015 school year, children will be exposed to it even earlier, in pre-K.

The pilot also demonstrates that an impactful curriculum need not be expensive or difficult to teach. The materials were all made by the teacher with foam core and velcro (for the class sized count-and-combine charts) and poster board (for the blocks used on the charts or for table work). Lesson plans (presented in one handbook) were designed to last a week, or longer. Progress depended entirely on what happened in the classroom, not on a set time-frame or a succession of daily work-sheets. In comparison to the two math curriculum she had previously used (*Everyday Math*© and the Scott Forseman/ Addison Wesley materials), the pilot teacher found the new curriculum easier (for her) to teach, and easier (for the children) to learn.

Creativity Models: Applications

With institution-wide problems, it is often easier for an "outside" expert to propose unconventional substitutions/solutions. It is why companies engage outside consultants to solve inside problems and, I presume, why I (an expert in problem solving per se, but not in mathematical problem solving) was asked to develop an early math curriculum. It is why I [along, I believe with Arthur Cropley (Shaughnessy, 2014b)] *strongly* urge other creativity researchers to direct their attention to educational applications. Our "outside" expertise can make a difference. The difference could be, would be, both significant and – in my experience - highly satisfying.

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[7] This does not mitigate the importance of inside-the-classroom expertise. I could not have translated my promote column into workable lesson plans without a highly skilled and enthusiastic kindergarten teacher as my partner (Thank you again, Mrs. Tronza). It also does not preclude mathematical expertise. I would not have seen how the blocks could be effectively used for multiplication and division (in the soon-to-be-introduced second grade curriculum) without the insights of a brilliant mathematician and friend (Thank you, Martin).

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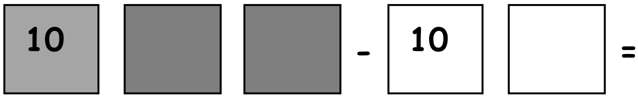
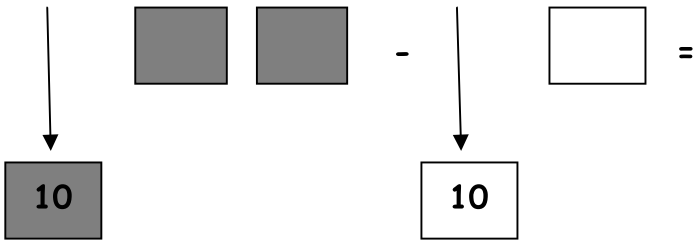
Key words: Paired constraints, Substitution, Mathematics, Place-value, Kindergarten

APPENDIX A: SUBTRACTING WITH ONES AND TENS

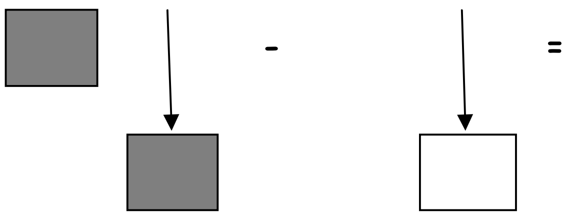
The children used red and green blocks for subtraction. The green blocks are shown in gray; the red blocks in white. The green blocks, placed on the left of the minus sign, represented the minuend. The red blocks, placed on the right side of the minus sign, represented the subtrahend, the number to be taken away. Children physically "took away" (one at a time) the same number of 10 and 1 blocks from both sides of the minus sign. For example, the problem 12 – 11 (ten-two minus ten-one) would look like this:



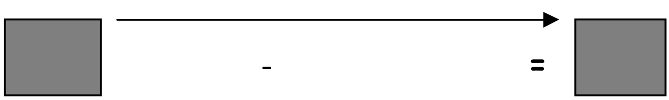
The children would first take away one red 10 block and one green 10 block.



They would then take away one red and one green 1 block.



When there were no red blocks left, they moved the remaining green block to the right side of the equals side.



They then wrote out (12 – 11 = 1) and recited (*one-ten-two minus one-ten-one equals one)* the solution.

APPENDIX B: MONTH-BY-MONTH INTRODUCTION TO NUMERICAL CONCEPTS IN PILOT AND CONTROL CLASSES.

Control Group: District curriculum

October-November:	Numbers through 5 Counting, Reading, Writing, Comparing
November-December:	Numbers through 10 Ordering, Ordinal numbers through 10^{th}
January-February:	Numbers through 31 Skip counting by 2s and 5s
May:	Readiness for addition and subtraction Ways to make 4 through 10 Understanding addition Using the plus sign/Finding the sum
June	Understanding subtraction Using the minus sign/Finding the difference Counting and number patterns to 100 Counting by 2s, 5s and 10s

Control Group: Explicit base-10 curriculum

September-October:	Count and combine (addition) numbers through 6
November:	Count and combine numbers through 10 Count and write numbers 11 to 20 (ten-one to two-ten) Recognize numeric patterns
December:	Count and write numbers through 30 (three-ten) Count by 10s to100 (ten to ten-ten) Introduce minus sign
January:	Count and combine numbers through 30 Count and write numbers to 40 (four-ten) Introduce the 10 block (see Fig. 3 and 5)
February-March:	Subtraction Count and write numbers through 50 (five-ten)
April-May:	Word problems Number line Double-digit addition and subtraction with 10-blocks